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A new Technique for Solving A Linear Programming Problem with Homogeneous constraints in Fuzzy Environment

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Abstract— This paper proposes a technique for solving a Fuzzy Linear Programming Problem(FLPP) when some of its constraints are fuzzy homogeneous. Using these fuzzy homogeneous constraints a fuzzy transformation matrix $\tilde{T}$ is constructed. This fuzzy transformation matrix $\tilde{T}$ transforms the given problem in to another FLPP with fewer fuzzy constraints. A relationship between these two problems, which ensures that the solution of the original problem can be recovered from the solution of the transformed problem. A simple numerical example explains the steps of the proposed techniques.

Index Terms— Triangular fuzzy number, Fuzzy homogeneous constraints and Fuzzy transformation matrix.

1 Introduction

The fuzzy set theory has been applied in many fields such as operations research, control theory and management sciences etc.. The fuzzy numbers and fuzzy values are widely used in engineering applications because of their suitability for representing uncertain information. Interval arithmetic was first suggested by Dwyer [5] in 1951. Development of interval arithmetic as a formal system and evidence of its value as a computational device was provided by Moore [16,17]. After this motivation and inspiration several authors such as Alefeld and Herzberger [1] Hansen[8,9,10], Luc. Jaulin et al [14], Lodwick and Jamison [13], Neumaier [20] etc., have studied interval arithmetic. The notion of Fuzzy number's as being convex and normal fuzzy set of some referential set was introduced by zadeh. The important contributions to theory of fuzzy number's and its applications have been made by many researchers. Some of the noteworthy contributions have been due to Dubois and Prade [3,4]. Kaufmann [11] , Kaufmann and Gupta [12] , Mizumoto and Tanaka [15] , Nahmias[19] and Nguyen [21].

The main aim of this paper is to reduce the computing time of the fuzzy optimization process when a block constraints are fuzzy homogeneous in nature. In this paper, section 2 gives some preliminaries which are very much needed for our article. Section 3 explains the fuzzy transformation matrix is constructed to solve the fuzzy linear programming problem with fuzzy homogeneous constraints. A relevant simple numerical example explains the proposed technique in section 4.Finally the conclusions are given in section 5.

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2 PRELIMINARIES

The preliminaries and function principle are studied through [18].

2.1.Fuzzy set

A fuzzy set $\tilde{A}$ is defined by

$\tilde{A} = \{(x, \mu_A(x)); x \in A, \mu_A(x) \in [0,1]\}$. In the pair $(x, \mu_A(x))$, the first element $x \in A$, the second element $\mu_A(x) \in [0,1]$ called membership function.

2.2.Support of fuzzy set

The support of fuzzy set $\tilde{A}$ is the set of all points x in X such that $\mu_{\tilde{A}}(x) > 0$. That is support $\tilde{A} = \{x / \mu_{\tilde{A}}(x) > 0\}$.

2.3. $\propto -cut$

The $\propto$- cut of $\propto$- level set of fuzzy set $\tilde{A}$ is a set consisting of those elements of the universe X whose membership values exceed the threshold level $\propto$. That is $\tilde{A_\propto} = \{x / \mu_{\tilde{A}}(x) \geq 0\}$.

2.4. Convex

A fuzzy set $\tilde{A}$ is convex if

$\mu_{\tilde{A}}(\lambda x_1 + (1 - \lambda x_2)) \geq mini(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2))$ $x_1, x_2 \in X$ and $\lambda \in [0,1]$. Alternatively a fuzzy set is convex if all $\propto$- level sets are convex.

2.5.Triangular fuzzy Number

It is a fuzzy number represented with three points as follows: $\tilde{A} = (a_1, a_2, a_3)$ this representation is interpreted as membership functions and holds the following conditions (i) a_1 to a_2 is increasing function.
(ii) a_2 to a_3 is decreasing function. (iii) $a_1 \leq a_2 \leq a_3$.

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & for \quad x < a_1 \\ \frac{x-a_1}{a_2-a_1} & for\ a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2} & for\ a_2 \leq x \leq a_3 \\ 0 & for\ x > a_2 \end{cases}$$

2.6. $\propto -cut$ of a triangular fuzzy number

We get a crisp interval by $\propto -cut$ operation, interval $A_\propto$ shall be obtained as follows for all $\propto \epsilon [0,1]$.

$A_\propto = [\propto_1^\propto, \propto_3^\propto]$
$(a_2 - a_1) \propto + a_1 ; (a_3 - a_2) \propto + a_3]$.

2.7.Positive triangular fuzzy number

A positive triangular fuzzy number $\tilde{A}$ is denoted as $\tilde{A} = (a_1, a_2, a_3)$ where all a_i's greater than zero.i = 1,2,3.

2.8. Negative triangular fuzzy number

A negative triangular fuzzy number $\tilde{A}$ is denoted as $\tilde{A} = (a_1, a_2, a_3)$ where all a_i's less than zero.i= 1,2,3.

Note: A negative triangular fuzzy number can be written as negative multiplication of a positive triangular fuzzy number.

2.9. Equal triangular fuzzy number

Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers. If $\tilde{A}$ is identically equal to $\tilde{B}$ only if $a_1 = b_1;\ a_2 = b_2;\ a_3 = b_3$.

2.10.Operations on triangular fuzzy numbers

Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers then

(i)Addition $\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$.

(ii)Subtrtion $\tilde{A} - \tilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$.

(iii)Multiplication

$$\tilde{A} X \tilde{B} = [mini(a_1b_1, a_1b_3, a_3b_1, , a_3\ b_3),\ a_2b_2,\ max(a_1b_1, a_1b_3, a_3b_1, , a_3\ b_3)]$$

(iv)Division

$$\tilde{A}/\tilde{B} = [mini(a_1/b_1, a_1/b_3, a_3/b_1,\ a_3\ /b_3),\ a_2/b_2,\ max(a_1/b_1, a_1/b_3, a_3/b_1,\ a_3/\ b_3)].$$

3 Fuzzy Linear Programming problem with Fuzzy Homogeneous Constraints

3.1. Definintion

A system of fuzzy linear constraints $\tilde{A}\tilde{X} = \tilde{b}$ is said to be fuzzy homogeneous constraints when $\tilde{b} = \tilde{0}$ such a system always have the trivial solution $\tilde{X} = \tilde{0}$.

3.2 Development of the fuzzy transformation matrix

In this section our objective is to develop a new technique to solve fuzzy linear programming problem with homogeneous constraints using fuzzy transformation matrix $\tilde{T}$.

Let the given FLPP be Maximize $\tilde{Z} = \tilde{C}x$ (3.1)
Subject to $\tilde{A}X = \tilde{b}$ (3.2)

and $\tilde{x} \geq \tilde{0}$. With $\tilde{a}_{i1}\tilde{x}_1 + \tilde{a}_{i2}\tilde{x}_2 + \tilde{a}_{i3}\tilde{x}_3 + \ldots + \tilde{a}_{ik}\tilde{x}_k + \ldots + \tilde{a}_{il}\tilde{x}_l + \ldots + \tilde{a}_{in}\tilde{x}_n = \tilde{0}$, for some i.

Here $\tilde{C} = (\tilde{c}_1, \tilde{c}_2, \tilde{c}_3, \ldots \tilde{c}_n)$ is a row vector with **n** fuzzy numbers , $\tilde{A} = (\tilde{a}_{ij})$ is a fuzzy matrix with **m** rows and **n** columns. Also $\tilde{a}_{ij}$, $\tilde{c}_i$ and $\tilde{b}_i$ are fuzzy numbers.We Partition fuzzy matrix as $\tilde{A} = (\tilde{A}^0, \tilde{A}^+, \tilde{A}^-)$. $\tilde{A}^0$ is the set of all column of $\tilde{A}$ whenever $\tilde{a}_{ij} = \tilde{0}$. Let **r** be the number of column $\tilde{A}^0$. $\tilde{A}^+$ is the set of all column of $\tilde{A}$ whenever $\tilde{a}_{ij} > \tilde{0}$. Let **p** be the number of column $\tilde{A}^+$. $\tilde{A}^-$ is the set of all column of $\tilde{A}$ whenever $\tilde{a}_{ij} < \tilde{0}$. Let **q** be the number of column $\tilde{A}^-$. Thus **p + q + r = n** which is order of fuzzy identity matrix . It is denoted by $\tilde{I}_n$. We define fuzzy transformation matrix $\tilde{T}$ as $\tilde{T}_{n\ X\ pq+r}$ such that the i[th] equation of $\tilde{A}\tilde{T}\tilde{w} = \tilde{b}$ will be $\tilde{0}$. Here $\tilde{w}$ is a column vector with pq+r components. This is accomplished by defining variables w_{kl} for each pair (k,l) such that $\tilde{A}_k \epsilon \tilde{A}^+$ and $\tilde{A}_l \epsilon \tilde{A}^-$. Now partition $\tilde{T} = (\tilde{T}_1 : \tilde{T}_2)$, where $\tilde{T}_1$ consist of unit fuzzy column vector $\tilde{e}_j$ corresponding to $\tilde{a}_{ij} = \tilde{0}$. $\tilde{T}_2$ consist of pq fuzzy column vector $\tilde{e}_j$ corresponding to w_{kl}. The fuzzy transformation matrix $\tilde{T}$ can be represented as

$\tilde{T} = [\ (\tilde{e}_j), \forall j \epsilon \tilde{a}_{ij} = \tilde{0}; (\tilde{t}_{kl}), \forall k \epsilon \tilde{A}^+, \forall l \epsilon \tilde{A}^-\ ]$. That is $(\tilde{e}_j)$ is the j[th] column of fuzzy identity matrix $\tilde{I}_n$ and $\tilde{t}_{kl} = -\tilde{a}_{il}\tilde{e}_k + \tilde{a}_{ik}\tilde{e}_l$.

Remark: Triangular Fuzzy representation of numberes $\tilde{0}$ and $\tilde{1}$ are (0,0,0) and(1,1,1) respectively.

4 Numerical example

Consider the triangular FLPP with homogeneous constraints as follows

Maximize $\tilde{Z}$ = $(1,2,3)\tilde{x}_1 + (2,3,4)\tilde{x}_2 + (9,10,11)\tilde{x}_3$ (4.1)

Subject to

$(1,1,1)\tilde{x}_1 + (0,0,0)\tilde{x}_2 + (1,2,3)\tilde{x}_3 = (0,0,0)$ (4.2)
$(0,0,0)\tilde{x}_1 + (1,1,1)\tilde{x}_2 + (1,1,1)\tilde{x}_3 = (1,1,1)$ (4.3)
and $\tilde{x}_1, \tilde{x}_2, \tilde{x}_3 \geq \tilde{0}$

Here the constraint equation (4.2) is a fuzzy homogeneous constraint, in which according to the section 3 we get p = 2 ,q = 0 and r = 1.

Therefore p + q+ r = 3 and pq + r = 1 , this implies order of $\tilde{T}$ is 3 X1

$$\tilde{I}_3 = \begin{bmatrix} (1,1,1) & (0,0,0) & (0,0,0) \\ (0,0,0) & (1,1,1) & (0,0,0) \\ (0,0,0) & (0,0,0) & (1,1,1) \end{bmatrix}$$

Here $\tilde{T} = \begin{bmatrix} (0,0,0) \\ (1,1,1) \\ (0,0,0) \end{bmatrix}$

From section 3, X=TW

Maximize $\tilde{Z} = \tilde{C}\tilde{T}\tilde{w}$ (4.4)

Subject to $\tilde{A}\tilde{T}\tilde{w} = \tilde{b}$ (4.5)

and $\tilde{w} \geq \tilde{0}$, we get
Maximize $\tilde{Z} = (2,3,4)\tilde{w}_2$ (4.6)

Subject to $(1,1,1)\tilde{w}_2 = (1,1,1)$ (4.7)
$\tilde{w}_2 \geq \tilde{0}$

The equation (4.7) implies $\tilde{w}_2$ = (1,1,1)

Therefore Maximize $\tilde{Z} = (2,3,4)$

Now solution of the original problem is $\tilde{X} = \tilde{T}\tilde{w}$

$$\begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix} = \begin{bmatrix} (0,0,0) \\ (1,1,1) \\ (0,0,0) \end{bmatrix} \tilde{w}_2 == \begin{bmatrix} (0,0,0) \\ \tilde{w}_2 \\ (0,0,0) \end{bmatrix}$$

This implies

Maximize $\tilde{Z} = (2,3,4)$ when
$\tilde{x}_1 = (0,0,0), \tilde{x}_2 = (1,1,1)$ and $\tilde{x}_3 = (0,0,0)$.

5 Conclusion

The technique explained in section 3, can be extended to define $\tilde{T}$ if $\tilde{A}\tilde{X} = \tilde{b}$ has more than one fuzzy homogeneous constraints, in case there are k fuzzy homogeneous constraints we define k fuzzy transformation matrices $\tilde{T}$(1), $\tilde{T}$(2), $\tilde{T}$(3) ,……… $\tilde{T}$(k). $\tilde{T}$(2) is determined once $\tilde{A}\tilde{T}$(1) has been computed. In general $\tilde{T}$(k)is determined only when $\tilde{A}\tilde{T}$(1), $\tilde{A}\tilde{T}$(2), $\tilde{A}\tilde{T}$(3)…… $\tilde{A}\tilde{T}$(k-1) has been computed. This technique reduces the number of constraints as well as computing times, the main factor of the optimization complexity, can be used efficiently for solving large-scale fuzzy programming problems. We can use this technique to fuzzy linear fractional programming problem with fuzzy homogeneous constraints.

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